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**HIGH ENERGY DENSE RIBBON RIBBON BEAMS AND
HIGH HARMONIC RECTANGULAR GYROTRON
AT MILLIMETER WAVELENGTHS**

by

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ABSTRACT

In this report, the progress made on the ribbon (sheet) beams and the rectangular harmonic gyrotron is presented. A relativistic 3-D electron gun simulation program, called 3-D_TRAJ is completed. A 3-D general numerical Poisson solvers is included in the program to take into account the nonuniform electrode boundaries that are common to ribbon beam guns or that may be required for any other gun structure. A novel computer technique is developed to generate the complicated 3-D general boundary data file of the MIG type guns by using a bot-map technique. At the same time, a theoretical approach to the design of MIG type guns is also undertaken. The location and the shape of the cathode is determined from the various conservation equations. Using the laminar flow equations and Harker synthesis technique, the required anode shapes are then determined. Finally, the resulting electrode shapes are used in the simulation of the MIG-gun using the 3-D_TRAJ program. Experimental set-up to test the various MIG type electron guns and the axially grooved rectangular gyrotron is close to being completed.

INTRODUCTION

High power millimeter waves are generated by conventional gyrotrons at the fundamental cyclotron frequency using an annular beam in a low Q cylindrical cavity /1/. Since the generated output frequency is directly proportional to the axial magnetic field, operation of the gyrotron at millimeter wavelengths require superconducting magnets to produce the very high magnetic fields.

Because of the quasi relativistic electron motion, the interaction between the electrons and the electromagnetic fields are also very efficient at the higher harmonics of the cyclotron frequency. Using a magnetron type gyromagnetron and a rotating pencil beam, it was suggested that the efficiency of higher harmonic operation can be further increased /2/. The power output of this mode of operation is limited by the large space charge forces present in the rotating pencil beam.

An alternate gyrotron configuration that uses a ribbon beam and an axially grooved rectangular waveguide, as shown in Figure 1, is proposed by Ferendeci /3/ with added advantages. Spreading the total current into a ribbon shaped beam reduces the space charge forces considerably. The use of the rectangular waveguide also reduces the thermal loading of the waveguide walls. The efficiency of coupling at the higher harmonics are also shown to be very high/4/. Recent numerical simulations of the axially grooved rectangular gyrotron verify that this interaction efficiency is indeed very high and the start oscillation current levels at the higher harmonics fall easily within the levels that are achievable with the present day electron beams /5/.

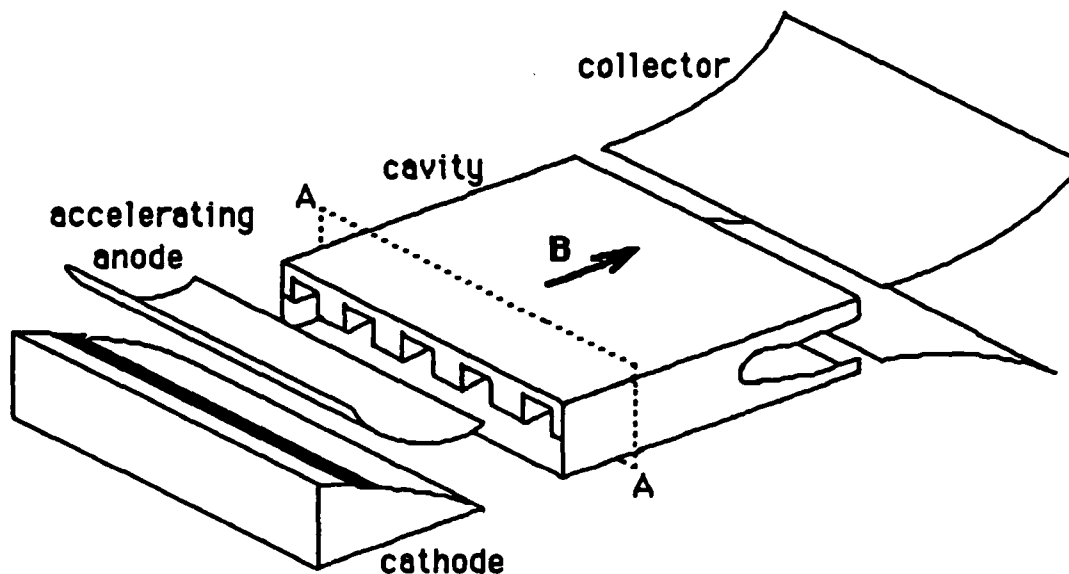


Figure 1. General schematic of the rectangular high harmonic gyrotron.

Ribbon beams, once successfully generated, will also find applications in other areas such as the quasi-optical cavity gyrotrons, free electron lasers, etc.

RIBBON BEAMS

Very elegant numerical synthesis techniques are developed in the design of cylindrical MIG guns/6,7/. Representing the electron beam by the temperature limited flow equations/8/, Harker electrode synthesis is used to determine the electrode structure consistent with the necessary laminar flow

of electrons/9/. This design procedure allows high power, low velocity spread electron beams to be generated in the MIG guns. By using the Herrmannsfeldt's SLAC-226 program/10/, the actual trajectories of the electrons and the spread in velocities are then calculated using the previously synthesized electrode geometry. Both the Herrmannsfeldt's code and the synthesis techniques incorporated in the design of rotating annular beams use cylindrically symmetric geometries. Although this technique can be used as an initial step in designing a ribbon beam, it does not predict the actual beam profile and characteristic of the ribbon beam.

THREE DIMENSIONAL TRAJECTORY (3-D TRAJ) PROGRAM

One of the fundamental difficulties in solving a three dimensional (3-D) trajectory problem lies in the solution of the Poisson (or Laplace) equation in three dimensions subject to the proper boundary conditions /11/. If the program is to be general in application and at the same time user friendly, provisions should be available in the program to enter any arbitrary shaped three dimensional boundaries into the program. This should also allow entry of the specific boundary conditions such as the Dirichlet and the Neumann boundaries. Another important difficulty in the solution of partial differential equations using finite difference techniques arises from the presence of the non-uniform mesh points that appear at these boundaries. If a generalized Poisson solver capable of overcoming the above difficulties is developed, calculation of the electron trajectories becomes a routine process.

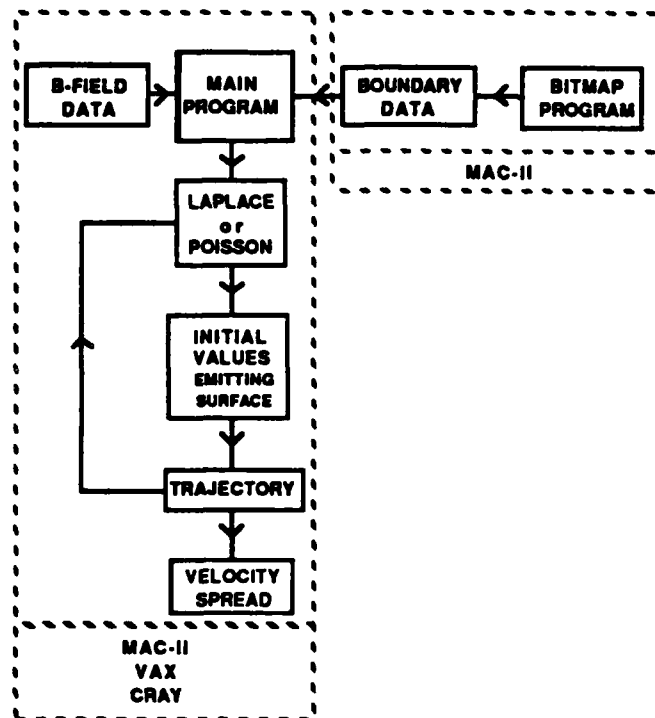


Figure 2. Block diagram of the 3-D_TRAJ Program

A generalized finite differences relativistic electron trajectory program (3-D_TRAJ) is now completed. The block diagram of the program is shown in Fig.2.

It consists of the following subprograms:

- 1) Boundary identification (BIT_MAP)
- 2) Poisson Solver,
- 3) Trajectory Solver

BOUNDARY IDENTIFICATION; BIT-MAP

One of the main difficulties in solving Poisson or Laplace equation is the determination of the fractional distances from the regular mesh points to the boundary locations.

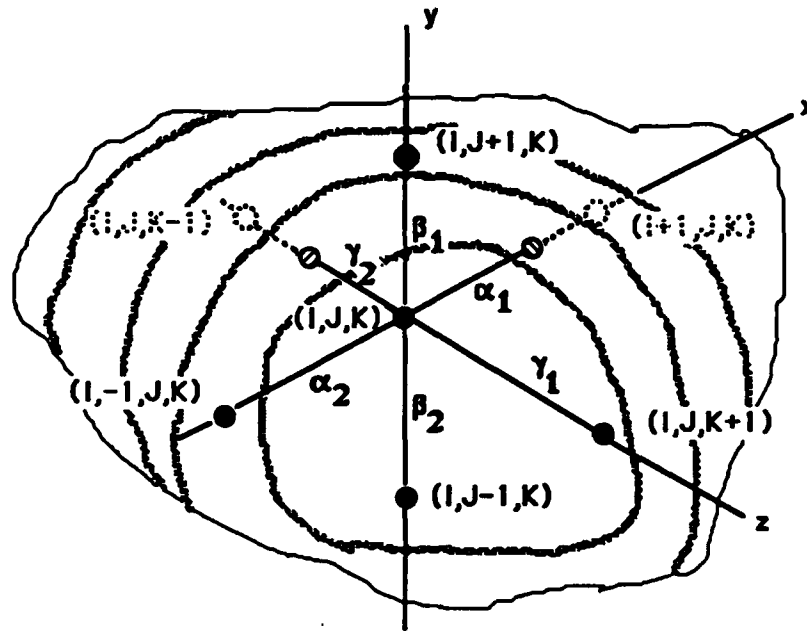


Figure 3. Fractional distances from the mesh point (i,j,k) to the boundary. For the case shown α_2 , γ_1 , β_1 and β_2 are equal to 1.0. α_1 and γ_2 are less than 1.0.

Fig.3 shows the various distances from the mesh point (i,j,k) to the immediate neighboring mesh points. The regular mesh distances in each direction are h_x , h_y and h_z . For the boundary given in the figure, the regular mesh points (i,j,k-1) and (i+1,j,k) fall inside a metal electrode. The corresponding Laplacian for the mesh point (i,j,k) can be written in terms of the general fractional distances as

$$\nabla^2 \Phi = \frac{2}{h_x^2(\alpha_1 + \alpha_2)} \left(\frac{\Phi_{i+1}}{\alpha_2} + \frac{\Phi_{i-1}}{\alpha_1} \right) + \frac{2}{h_y^2(\beta_1 + \beta_2)} \left(\frac{\Phi_{j+1}}{\beta_2} + \frac{\Phi_{j-1}}{\beta_1} \right) + \frac{2}{h_z^2(\gamma_1 + \gamma_2)} \left(\frac{\Phi_{k+1}}{\gamma_2} + \frac{\Phi_{k-1}}{\gamma_1} \right) - \left[\frac{2}{h_x^2 \alpha_1 \alpha_2} + \frac{2}{h_y^2 \beta_1 \beta_2} + \frac{2}{h_z^2 \gamma_1 \gamma_2} \right] \Phi$$

Here the fractional distances α_1 and α_2 , β_1 and β_2 and γ_1 and γ_2 are in the respective plus and minus x, y and z directions. In the above equation, it is implied that the potentials include integers (i, j and/or k) if they are not explicitly written. For the geometry shown in Fig.3, the fractional distances α_2 , γ_1 , β_1 and β_2 are equal to 1.0. and α_1 and γ_2 are less than 1.0.

In order to determine the fractional distances, a computer generated approach using a bit-map technique is used.

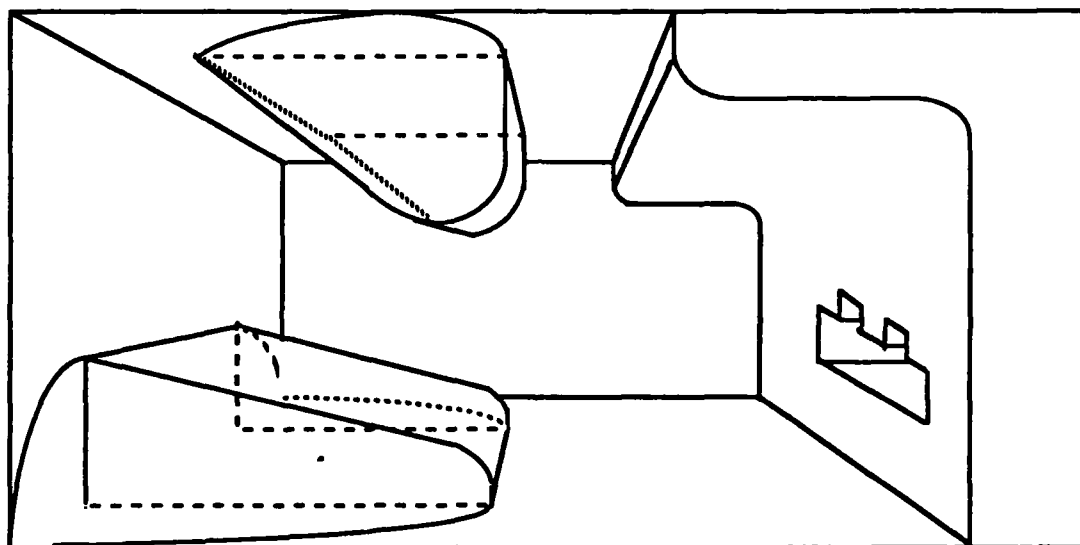


Figure 4. Perspective drawing of a typical MIG gun used to test the numerical code.

Figure 4 shows a typical rectangular gyrotron. It has a cathode, and two anodes. Fractional distances for electrodes that can be represented by simple surfaces can be calculated relatively easily. In general, many portions of the electrodes can not easily be represented by analytical expressions. This becomes especially difficult when unusual accelerator electrode shapes are required.

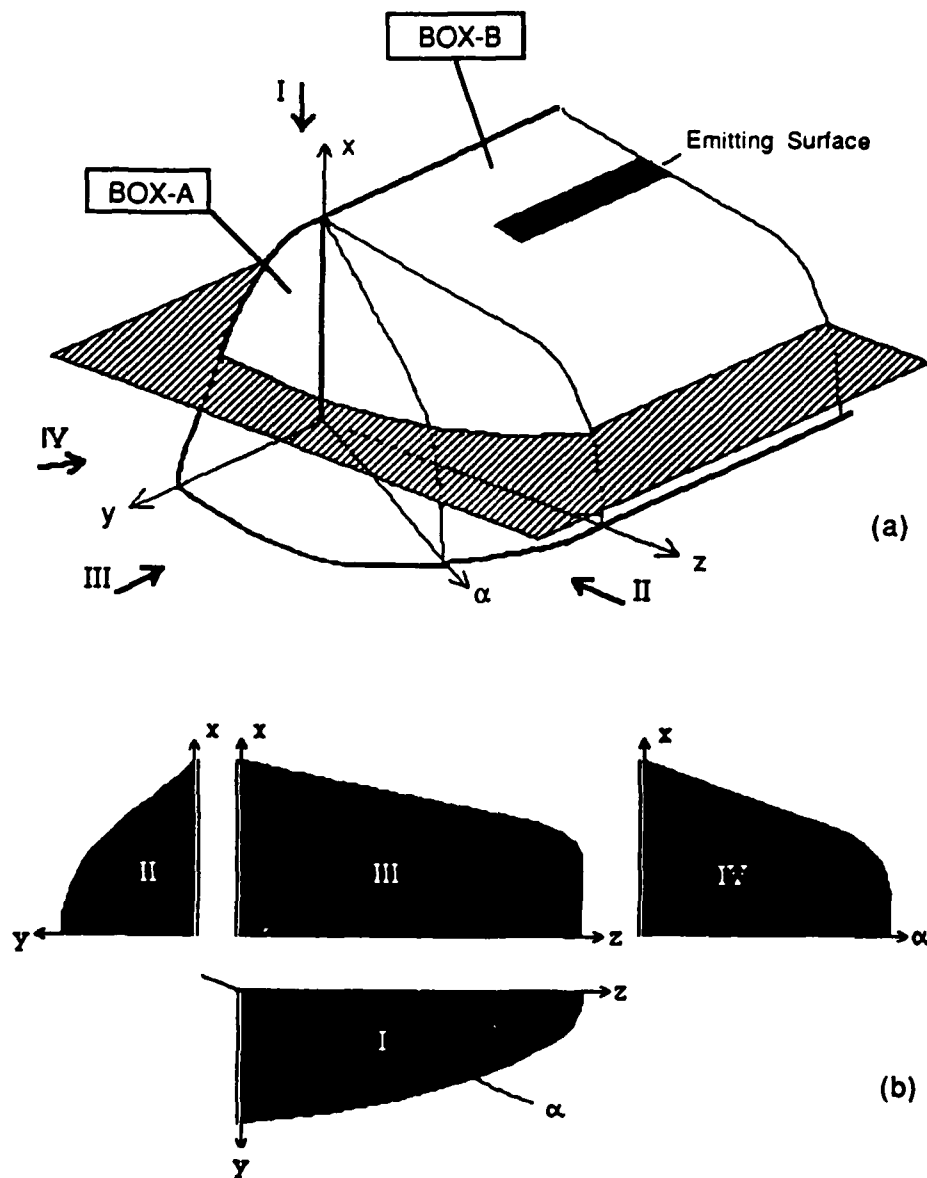


Figure 5. a) Two boxes of a typical electrode and b) three representative sections of the corner of the cathode. The lower cross section is redundant

To calculate the fractional distances, the following procedure is used. A given electrode is divided into smaller boxes with the requirements that any cross sectional plane within this box can be approximated by a quadratic

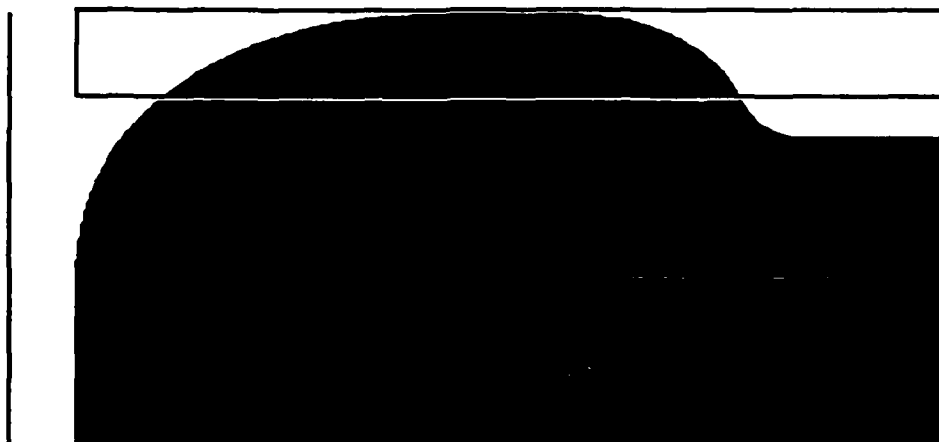
equation. Fig.5 shows two adjacent boxes associated with the cathode. In order to calculate the fractional distances for BOX-A, three representative cross sections are chosen as shown in Fig.5.b. Only one cross section is required for BOX-B.

These cross sections are drawn to scale as MacPaint documents on a Macintosh computer. They are then bit-mapped using a simple program/12/. Fig.6 shows a typical cross section and the corresponding bit-map for the upper (boxed) region of a sample plane with more complicated cross section.

Within the chosen accuracy of the bit-map program, once the locations of the points corresponding to these three plane boundaries are calculated, a 2nd order curve fitting passing through the three boundary points at a given j-plane is used to calculate Δx and Δz . Using this data, all Δy 's are then calculated for the given box. The resulting Deltas are stored in a file DELTAS(DIR,i,j,k) where DIR=x,y,z.

All the electrodes of the gun are assigned a number. Each numbered electrode is also identified with the corresponding potential for that electrode. The cathode is 5.0 and at $\Phi=0.0$ V. The anodes are numbered in such a way that one can have 10 different electrodes (numbered from 11.0 to 20.0) with different potentials. (Actually there is no limit to the number of electrodes one can use in the program). 2.0 is reserved for the Neumann boundary. Figure 6 shows the Δz of the gun at the cathode region at the plane $i=40$ for $j=1 \rightarrow 25$ and $k=1 \rightarrow 30$. The (*) represent numbers >10.0 .

Once the boundary file is created, the program is either run on MacII using this data file, or transferred to a VAX or a CRAY for more faster and efficient calculations.



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1 100000000000000000000003FFFFFFFFFFFFFFFF000000000000
2 100000000000000000000000FFFFFFFFFFFFFFFFFFFFE0000000000
3 100000000000000000000007FFFFFFFFFFFFFFFFFFFFF80000000000
4 100000000000000000000001FFFFFFFFFFFFFFFFFFFFF80000000000
5 100000000000000000000001FFFFFFFFFFFFFFFFFFFFFE0000000000
6 100000000000000000000001FFFFFFFFFFFFFFFFFFFFFC0000000000
7 100000000000000000000000FFFFFFFFFFFFFFFFFFFFFFFF000000000
8 100000000000000000000007FFFFFFFFFFFFFFFFFFFFFC000000000
9 100000000000000000000001FFFFFFFFFFFFFFFFFFFFFFFF000000000
10 100000000000000000000000FFFFFFFFFFFFFFFFFFFFFC000000000
11 100000000000000000000003FFFFFFFFFFFFFFFFFFFFFE000000000
12 100000000000000000000001FFFFFFFFFFFFFFFFFFFFFFFF800000000
13 100000000000000000000007FFFFFFFFFFFFFFFFFFFFFC000000000
14 100000000000000000000001FFFFFFFFFFFFFFFFFFFFFE000000000
15 100000000000000000000007FFFFFFFFFFFFFFFFFFFFFFFF000000000
16 10000000000000000000000FFFFFFFFFFFFFFFFFFFFC0000000000
17 100000000000003FFFFFFFFFFFFFFFFFFFFFFFFFFFFFE0000000000
18 100000000000000FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF000000000
19 10000000000001FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF8000000000
20 10000000000007FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFC000000000
21 100000000000FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF0000000000
22 1000000000003FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFE0000000000
23 1000000000007FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF0000000000
24 1000000000001FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF8000000000
25 1000000000003FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFC0000000000
26 1000000000007FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFC0000000000
27 1000000000001FFFFFFFFFFFFF1FFFFFFFFFFFFFFFFFFFFFFFFFE0000000
28 1000000000003FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF0000000000
29 1000000000007FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF0000000000
30 1000000000000FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF8000000000

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Figure 6. A scaled cross section of a typical electrode and b) bit-mapped output of the boxed section.

MAGNETIC FIELD DATA

Magnetic field data can be inputted in various ways:

- a) Constant B_z ,
- b) Increasing B_z ,
- c) Data from a File,
- d) Calculate B from Biot-Savart Law,
- e) Calculate B through elliptic integrals.

In cases c→e above, there is also a subprogram to Convert the magnetic field data from Cylindrical (B_r, B_z) to Rectangular (B_x, B_y, B_z) components. The coordinate center of a magnetic field data can be adjusted with respect to the gyrotron axis by specifying the distances x_c and y_c as shown in Fig.7. This is necessary to take into account the displacements of the electrons due to the drift at the cathode region.

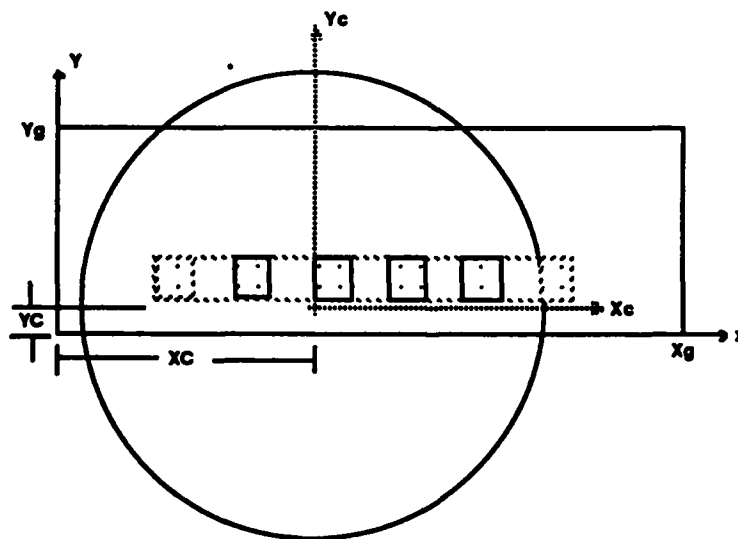


Figure 7. The location of the Magnetic field coil can be changed with respect to the gyrotron coordinate system.

MAIN PROGRAM

Once the boundary file is created and the magnetic field data is generated, the main program takes over as shown in the Block diagram of Fig.2.

First, Laplace equation is solved by setting the right hand side of the Poisson equation to zero. An over-relaxation method is used for the iteration solution of the Poisson equation. Either a minimum residue is set or, for the initial calculations, a maximum number of iterations is chosen. At the conclusion of the Laplace solution, there is an option to plot the equipotential lines in two dimensions. Any errors in the fractional distances, i.e., file DELTAS, show as unusual potential values in the vicinity of the error points. A three dimensional equipotential profile with correct calculated fractional distances is shown in Fig.8.

Once the potential data is found to be satisfactory, the solution continues with the location of the emitting surface and calculation of the initial velocities of the beamlets. The maximum number of rays can be set at 1000 but this requires extensive memory space and at the same time slows down the execution time. To prevent this, the maximum number of rays is limited to 300 in the development of the program.

Next, the trajectory calculations using a fourth order Runge-Kutta method is initiated. After a calculation is made for a ray, the charge corresponding for that ray at that point is stored in the neighboring mesh points in proportion to their distances from the location of the charge. During the trajectory run, the electron trajectories are also plotted in any chosen plane. Once the program debugging is completed, these plotting routines can

be omitted to increase the execution time. Figure 9 shows typical trajectory plots at the (x,y) , (y,z) and (x,z) planes.

After the initial iteration process described above, the Poisson equation is solved next this time taking into account the charge stored at each mesh point during the previous trajectory calculation. Initial conditions for each ray are then recalculated and the new trajectory calculations are made using the new potentials. "Poisson+initial conditions+trajectories" constitute one iteration for the main loop in Fig.2. The total number of main-iterations can be set during the initial program data input.

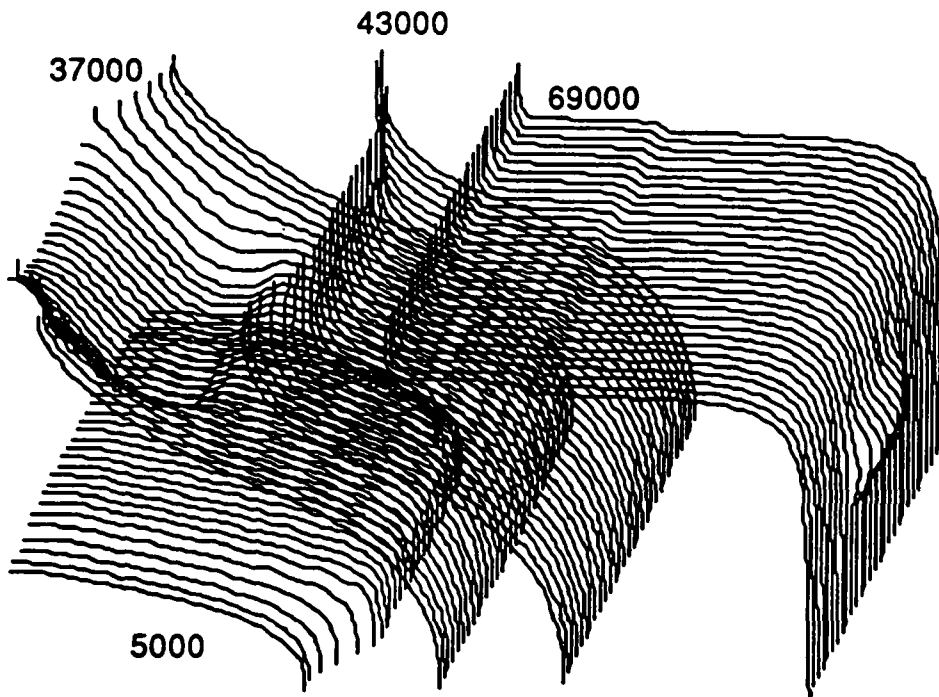


Figure 8 Three dimensional plot of the equipotential surfaces

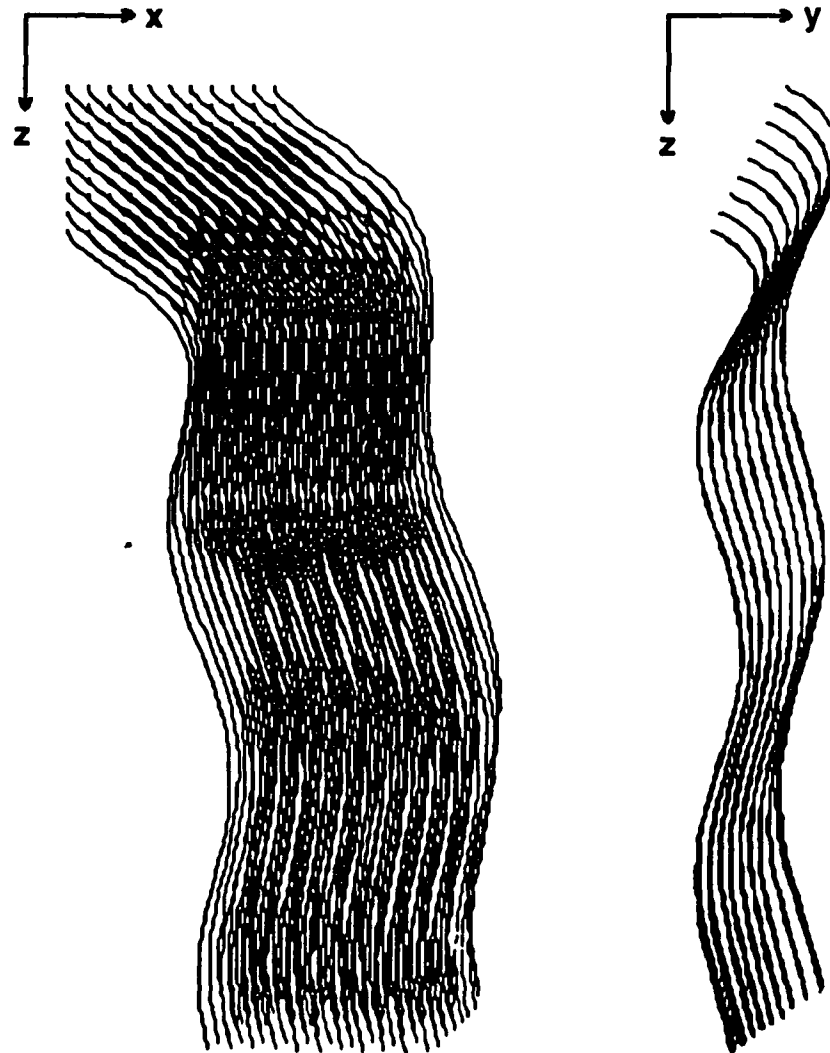
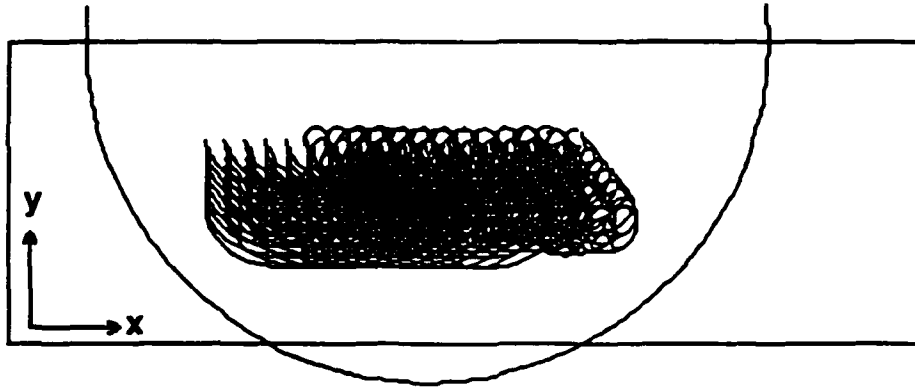


Figure 9. The trajectory plots after an iteration in (y,x) , (z,x)
and (z,y) planes.

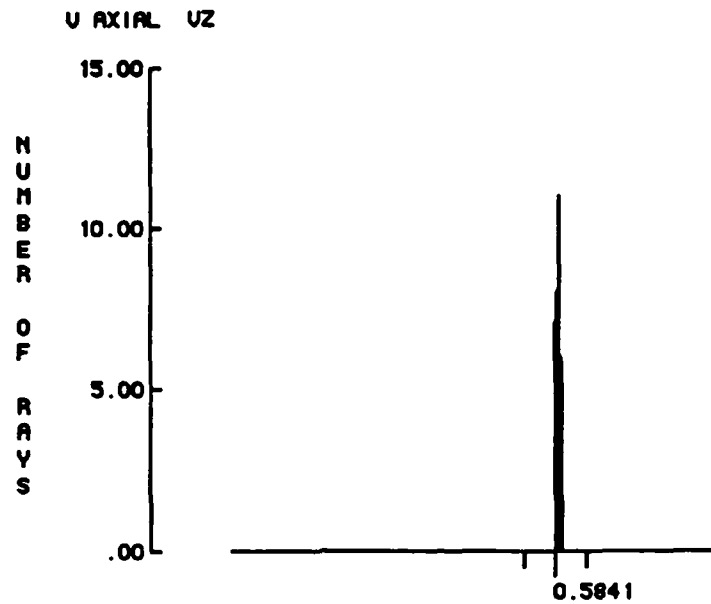


Figure 10.a. Sample plot of axial velocity distribution of electrons (# of rays=100). The average v_z and $\pm 10\%$ v_z are indicated in the figure.

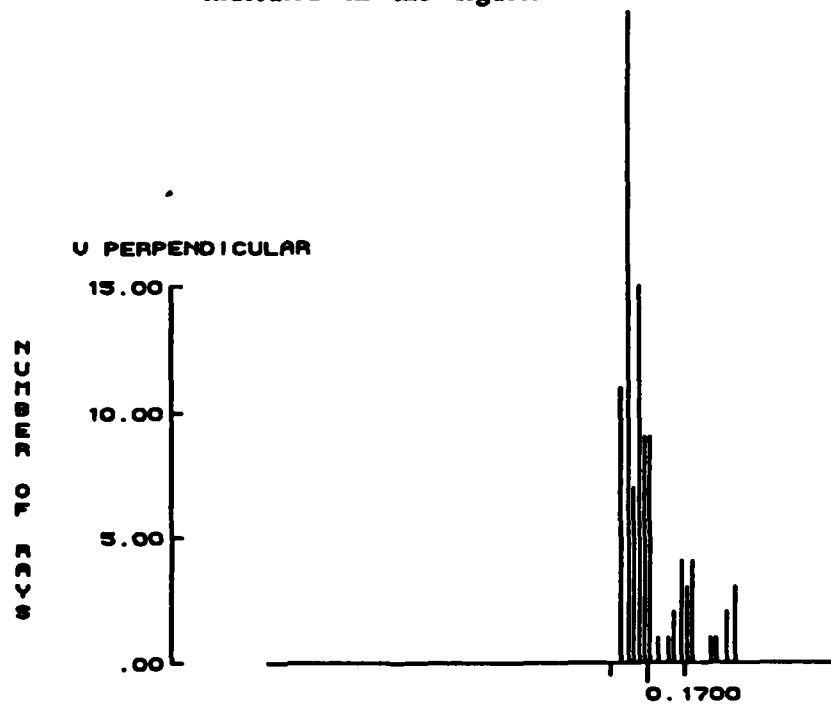


Figure 10.b. Sample plot of perpendicular velocity distribution of electrons (# of rays=100). The average v_{\perp} and $\pm 10\%$ v_{\perp} are indicated in the figure.

The final results contain various useful data. In addition to the current per each ray, the perveance of the gun, the final axial and perpendicular velocities and their final x,y,z locations for each ray can be stored in a file. One can also calculate the distribution of the final axial and perpendicular velocities and display them in graphical form as shown in Fig.10.a and b. $\pm 10\%$ deviations from the average velocities are also shown in the same figures to indicate the extent of the velocity spread of the electrons.

In its present form, the 3-D_TRAJ program can:

- a) Solve Laplace or Poisson equation with arbitrary electrode geometries.
- b) Calculate trajectories of relativistic electrons,
- c) Calculate the perveance of the gun, and
- d) Can use any type of magnetic field data

At its present form, the program does not:

- a) Include self-magnetic fields,
- b) Check for beam crossings and
- c) Include thermal effects.

One of the limiting aspects of the program is the choice of the number of mesh points which critically depend on the RAM memory space available for storing the various array information. For a choice of NXMAX=80, NYMAX=40 and NZMAX=90 mesh points, the following arrays are needed:

DELTAS(3,80,40,90)

POT(80,40,90)

RAYS_INFO(9,300)

BFIELD(3,19,19,50)

For single precision and 2'nd order curve fitting, to run the program with these arrays, ≈ 5.0 Mb of RAM memory is required. The accuracy of the program can be increased by

- Increasing the number of mesh points,
- Using double precision
- Using higher order Runge-Kutta and
- Using higher order curve fitting

at the expense of increasing the execution time and the memory requirements of the computer.

GUN DESIGN

Gun design for a ribbon (sheet) beam to be used in an axially grooved rectangular gyrotron has more stringent requirements compared to other gyrotrons, such as the quasi-optical gyrotron which also requires a sheet beam for more efficient operation. As shown in Fig.11, the ideal height h of the electron beam at the gyrotron location should be equal to $2\rho_L$. But, because of the extended height L_z of the cathode, the thickness of the rotating e-beam becomes greater than $2\rho_L$ as the beam enters the interaction region of the gyrotron.

The beam requirements can be summarized as follows:

- i) the thickness h of the beam should not exceed the cavity height b ,
- ii) the cross section of the beam should be as close to a rectangular shape as possible to prevent beam interception by the cavity walls and,
- iii) beam should have minimum spread in perpendicular velocity as it enters the interaction region for maximum efficiency.

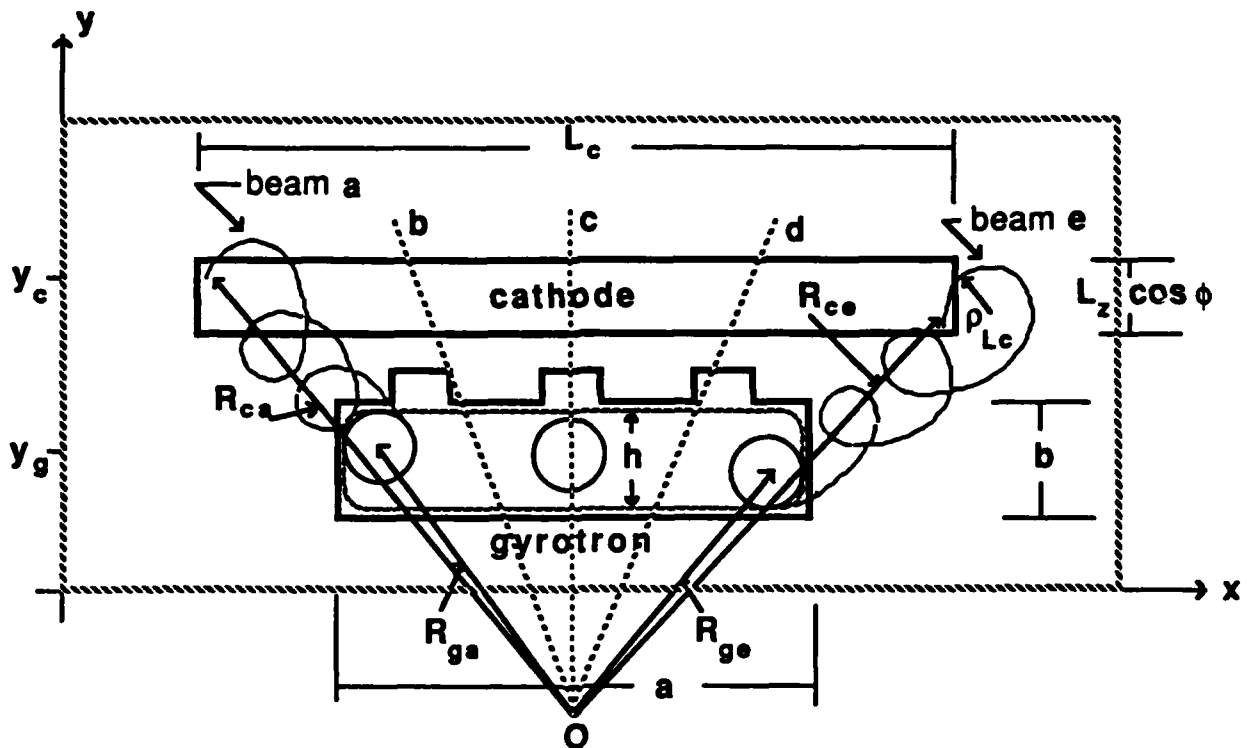


Figure 11. (x,y) projection of the MIG gun for generating ribbon beams.

Since the electrons originate from different geometrical locations at the cathode, their origin of emission should be such that they end up within the gyrotron interaction region and not intercepted by the walls. Respective beams, *a* originating from the top left corner and *e* originating from the right lower corner of the cathode, should end up at the upper left and lower right corners of the gyrotron. The initial electron guiding center displacements should also be taken into account. The actual magnetic field lines that the electrons are expected to follow will not coincide with field lines at their origins of emission but will be displaced by their respective electron Larmor radii.

The spread in v_{\perp} as shown in Fig.10.b for a flat cathode has +30% spread in v_{\perp} which will not be acceptable for the efficient operation of the gyrotron. Thus the gun design should produce minimal spread in this velocity.

In order to satisfy the above requirements, a novel design approach is used to find the proper electrode shapes for a rectangular version of a MIG gun to produce an acceptable rotating sheet beam. Because of the electrode configuration of the gyrotron and the magnetic field used in realizing this gyrotron, both rectangular and cylindrical coordinates are carefully incorporated into the design procedure. The procedure also requires, as a priori knowledge, the spatial variation of the d.c. magnetic field produced by the actual solenoidal coil that is used in the experiment.

Fig.11 shows the projections of the cathode and the gyrotron at the (x,y) plane. The ideal location of the emitting surface has to be found in such a way that all the electrons entering the interaction region will all have the same v_{\perp} as well as the same velocity ratio $\alpha = v_{\perp}/v_z$.

In the design of the gun, following equations are used /13/:

i) From the conservation of conanical momentum

$$B_c y_c = B_g y_g \quad (1)$$

2) Larmor radius at the cathode

$$\rho_L = \frac{m E_c}{e B_c^2} \quad (2)$$

3) Velocity ratio

$$\alpha = \left(\frac{B_g}{B_c} \right)^{1/2} \frac{E_c}{\sqrt{2 \frac{e}{m} V_0 - E_c^2 \left(\frac{B_g}{B_c} \right)}} \quad (3)$$

Here B_c = magnetic field at the cathode

B_g = magnetic field at the gyrotron

y_c = guiding center at the cathode

y_g = guiding center at the gyrotron

E_c = electric field at the cathode

V_o = final accelerating voltage.

The maximum drift is assumed to occur at the cathode and thus, the electrons emitted at the cathode are assumed to follow the magnetic field lines shifted by the Larmor radii of the electrons with respect to their origins of emission. The cathode electric field E_c is also be assumed to be known.

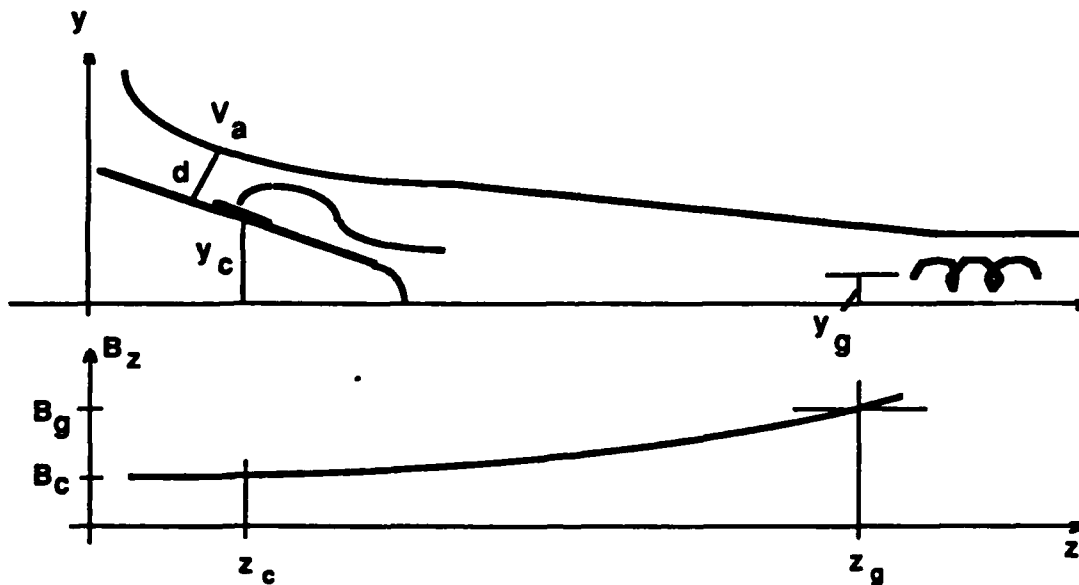


Figure 12. The (y,z) plane of the gun and the location of the corresponding parameters.

The magnetic fields generated by a conventional solenoidal coil has cylindrical symmetry. For a given electron guiding center y_g at the gyrotron, the corresponding magnetic field B_g at the same location is found from the magnetic field data by using a simple computer program. For a compression

ratio α (1.3 for the sample design) and a final accelerating potential of V_0 , the cathode magnetic field necessary to give this compression ratio is found by calculating the ratio (B_g/B_c) from Eq.3. Then using Eq.1, y_c is calculated.

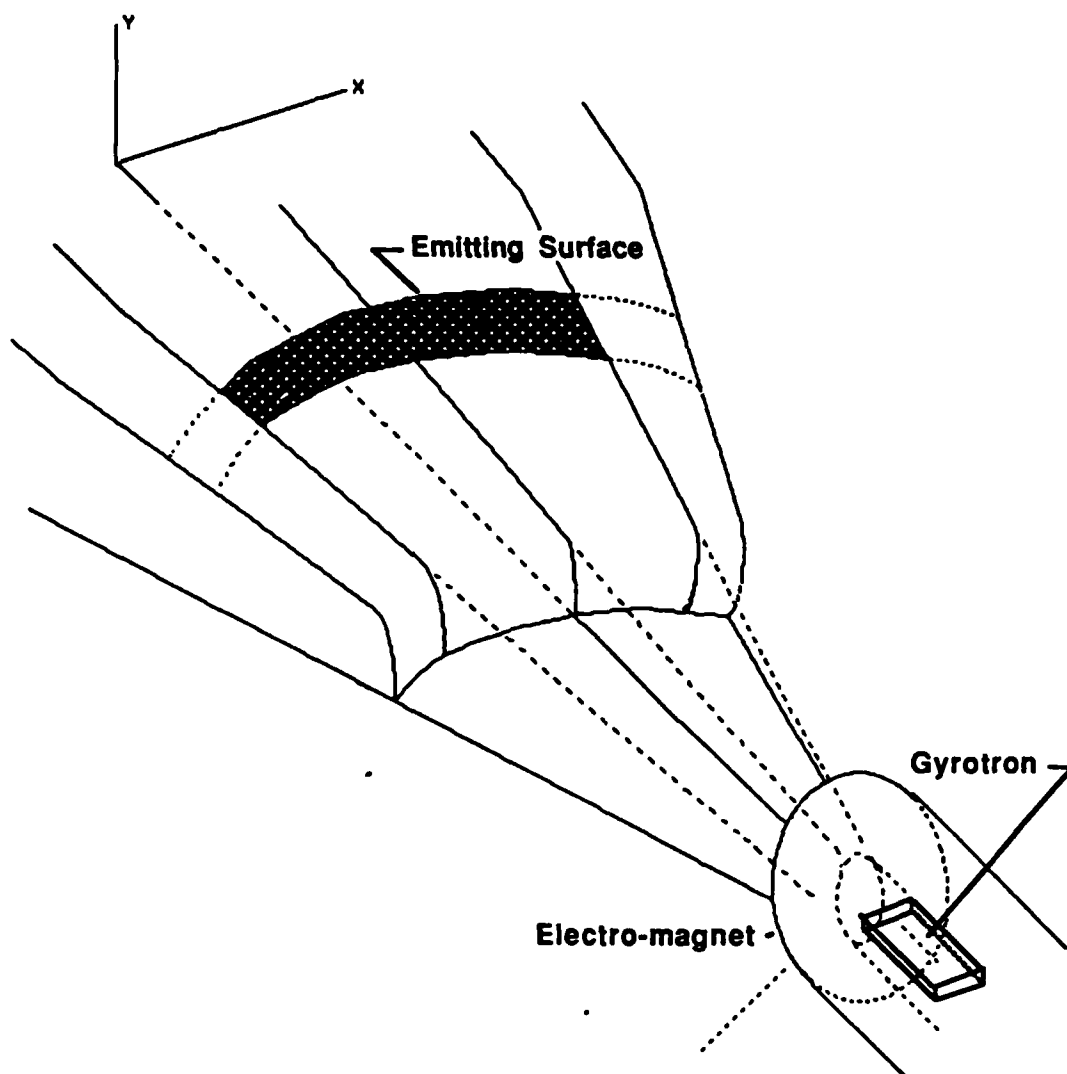


Figure 13. Emitting surface for $V_0=80$ kV and $\alpha=1.3$.

The magnetic field data is then scanned at the plane y_c . The corresponding axial distance z_c where the magnetic field is equal to B_c is found

(Fig.12). The actual emitted surface is then found by translating the guiding center by the Larmor radius of the electron at the cathode using Eq.2.

In principle, this procedure is repeated for the number of electron beamlets by incrementing the electron guiding center locations at the gyrotron. To save calculational time, only representative electron beamlets in the planes a, c and e are used in the final cathode surface plot given in Fig.13.

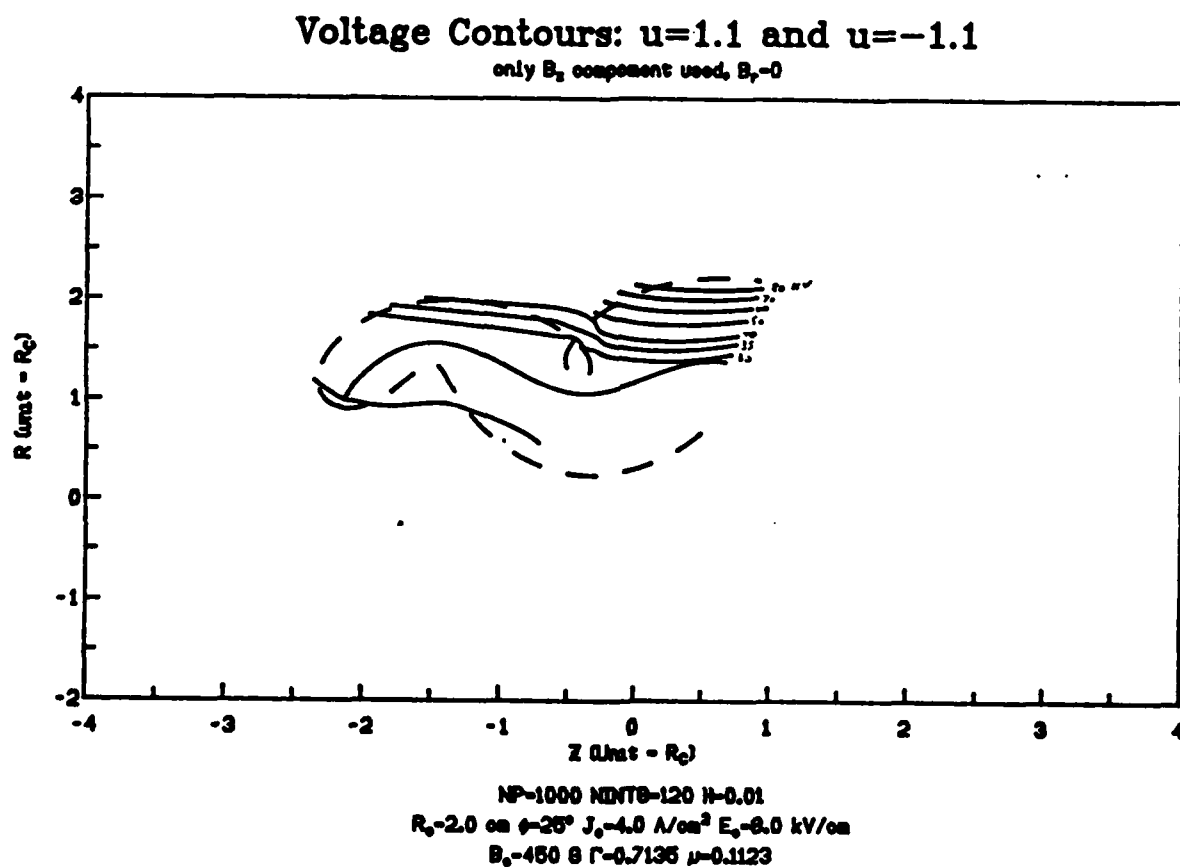


Figure 14. Potential profile at the plane c for the rectangular MIG gun for $E_c=8$ kV/cm and $J=4$ A/cm², $R_c=2.0$ cm. Range is limited for small R_c .

Voltage Contours: $u=1.2$ and $u=-1.2$

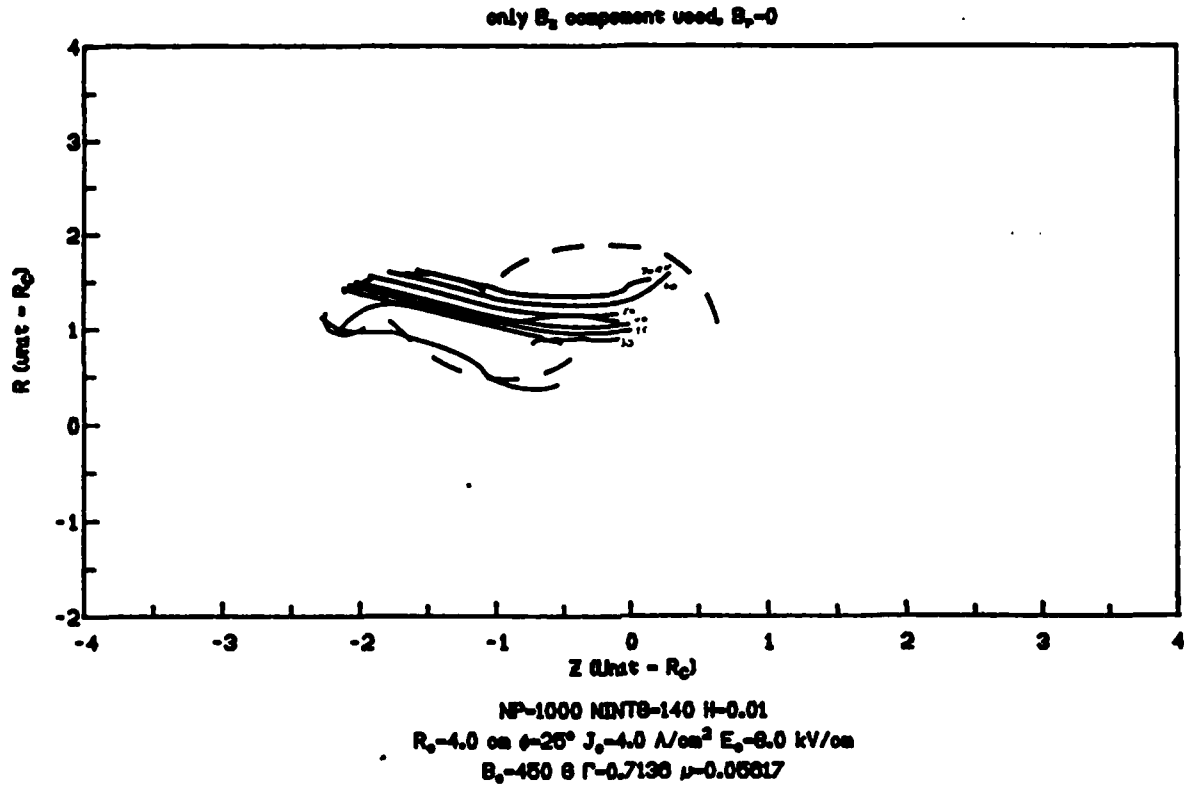


Figure 15. Potential profiles at the plane c for the rectangular MIG gun for $E_c = 8$ kV/cm and $J = 4$ A/cm², $R_c = 4.0$ cm.

The next step is to find the corresponding anode electrode shapes which will produce the assumed electric field at the cathode, but at the same time produce the necessary electron flow conditions for the beam. For the combination of the cylindrically symmetric magnetic field and the rectangular gun geometry, no electrode synthesis technique is available at present. Harker type synthesis technique, developed for the cylindrically symmetric MIG guns, can not be used for the given geometry because of the

resulting angular component E_θ of E_c . But at the plane c, the electric field is purely radial and to a first order approximation, the gun can be synthesized at this plane using the Harker synthesis technique. A computer program following Ref.(6, 7) is written to simulate the gun at this plane. The results of this simulation are shown in Fig. 14 & 15. For the low magnetic fields used in the design of the gun ($B_g=0.5$ Tesla), the range of the useful solution becomes limited for high cathode electric fields and high current densities. Reducing E_c allows increase in the corresponding range of the applicable equipotentials.

EXPERIMENT

An experiment set-up to test the ribbon beam and the rectangular gyrotron is close to being completed. For the initial experiments, special planar cathodes as shown in Fig.16.a are obtained from Spectra-Mat, Inc. They are rated at 10 A of total current and their emitting areas are 2 mm by 20 mm. The cathode electrode is at present is being machined. The cathode will be inserted in a slot machined through the cathode electrode at the proper angle as shown in Fig.16.b. Once the system is evacuated, the cathode will be activated and initial runs will be carried out.

The general gyrotron experimental set-up is shown in Fig.17. A Vac-Ion vacuum pump and stainless steel tubing with Con-Flat flanges are used. The roughing pump is a cryogenic Vac-Sorb pump. The system is capable of producing pressures less than 10^{-8} Torr.

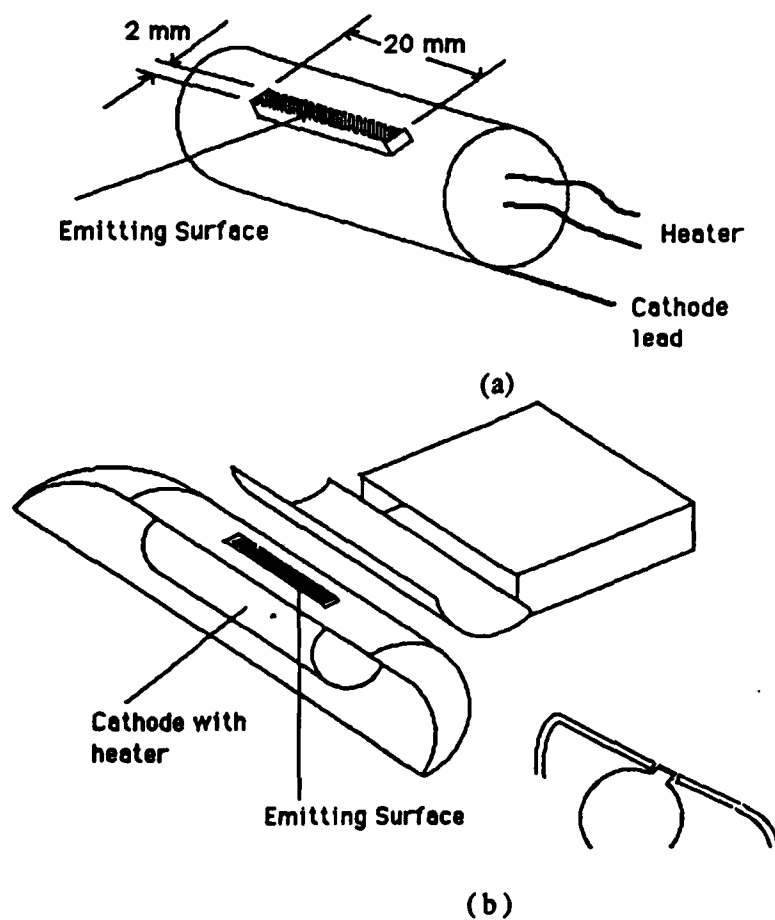


Figure 16. a) the cathode and the heater assembly and b) location of emitting element in the cathode electrode.

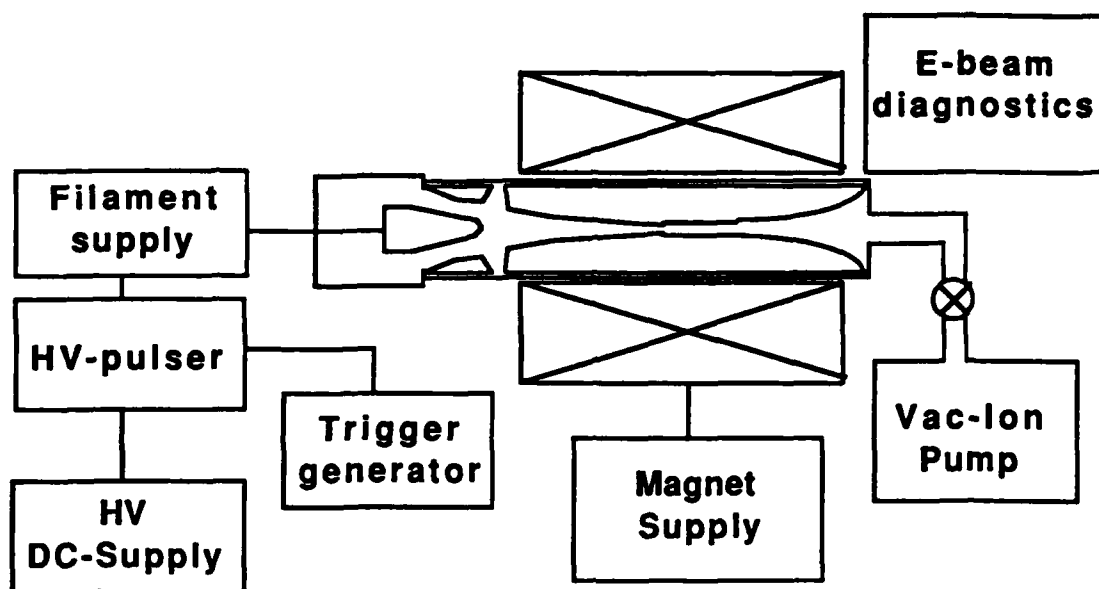


Figure 17. Vacuum system and the associated electronic system.

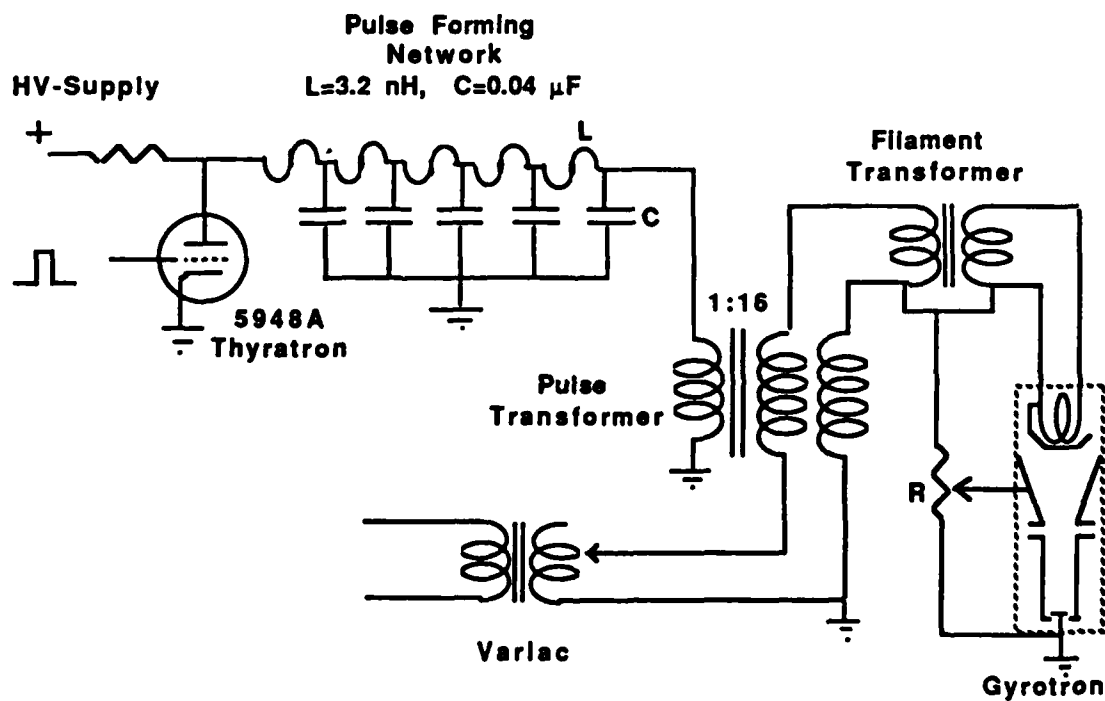


Figure 18. Essential components of the pulse circuit for the gyrotron

A high voltage pulse generator capable of producing 80 kV pulse with 0.1 μ sec rise time, 3 μ sec pulse duration, repetition rate of 4 pps and a current range of 2-40 A is designed (Fig.18). The circuit is similar to the MIT design /14/ and the pulse transformer containing the bifilar winding is manufactured by Stangenes, Inc., CA.

PUBLICATIONS

Various papers on the design of the MIG-gun and on the development of 3-D_TRAJ program are presented at the International Conferences on Infrared and Millimeter Waves and APS Plasma Physics Conference. Copies of the abstracts presented in these conferences are attached as an appendix. Two Masters Thesis are completed, One, related to the 3-D Pierce type gun (Student: Djamal Sulimani /15/), is completed in December of 1987. A second Master's thesis (Student: Ammar Darkazanli /16/) related to the generalized Poisson solver and the MIG gun is also completed in April, 1988. A Ph.D. student, Kurt Ericksen is at present involved with the experimental and theoretical parts of the research work at UC.

A paper entitled *"Numerical Solution of Laplace Equation in Three Dimensions for a Pierce type Electron Gun"* is already published in Computers & Electrical Engineering, Vol.15, No.3/4, pp.97-105 (1989). A Two additional papers, one on *"3-D_TRAJ computer Program"* and the other on the *"Resonant Modes of An Axially Grooved Rectangular Cavity"* are being prepared and will be submitted shortly for publication

As a result of accepting a new position at the Electrical and Computer Engineering Department of the University of Cincinnati, progress in the research project was initially slowed down considerably during the 1988-1989

academic year. But now with the addition of Kurt Ericksen into the program, the research project is moving along in an accelerated pace.

CONCLUSIONS

Most of the work done previously was concentrated on the design of rectangular versions of the MIG gun. Various numerical programs have now been developed to simulate such a gun. A general three dimensional relativistic electron trajectory program called 3-D_TRAJ is now completed. This program, which can be thought of as a three dimensional extension of Herrmannsfeldt's SLAC-260 program, is capable of solving Poisson equation with arbitrary geometries. Certain programs run using this program indicate that ribbon (sheet) beam gun designs based on Herrmannsfeldt's cylindrically symmetric code introduces considerable velocity spread in the actual electron velocities.

Although the experimental progress has been slowed down due to the change in position from Case Western Reserve University to the University of Cincinnati, the gap is now closed due to the availability of better equipment and facilities at the University of Cincinnati.

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- /15/ Djamal Slumani, "Three Dimensional Computer Simulation of the Potential Field in an Electron Gun.", M.S.Thesis, Case Western Reserve University, (1987).

/16/ Ammar Darkazanli, "Three Dimensional Simulation of Electron Beam Gun", M.S.Thesis, Case Western Reserve University, May 1988.

APPENDIX

Copies of abstracts and papers presented at various conferences and printed in journals.

**Design and Operation
of
Ribbon Beams for Gyrotrons**

**Altan M. Ferendeci
ECE Dept., University of Cincinnati
Cincinnati, Ohio 45221**

&

**Ammar Darkazanli
EEAP Dept.
Case Western Reserve University
Cleveland, Ohio 44106**

Design of a modified three dimensional MIG gun to produce energetic rotating electron ribbon beams are presented. Two dimensional synthesis technique is used to find the best possible electrode configuration in the central plane. The gun is simulated using a 3-D numerical code that has been developed to take into account the possible electrode geometries that are encountered in such guns. Compression of the beam in one dimensions shows marked improvements over compression in two dimensions which pose certain restrictions on the performance of the gun. A gun is being constructed to test the results.

Presented at the "13'th International Conference on Infrared and Millimeter Waves", SPIE, Vol.1039, Paper F.8.12. Honolulu, Hawaii, 1988.

Abstract submitted for the Thirty-first Annual Meeting
Division of Plasma Physics
November 13-17 1989

Category Number and Subject _____

☐ Theory ☐ Experiment

Synthesis and Design of Sheet Beams for Gyrotrons. ALTAN M. FERENDECI and K. ERICKSEN, University of Cincinnati.-- Design of high energy sheet beams to be used in a high harmonic rectangular gyrotron will be presented. A synthesis technique taking into account the spatial variation of the magnetic field as well as the radial component of the magnetic field is used to find the proper electrode shapes simulating laminar flow of the beam. The resulting shapes are then used in the simulation of a three dimensional trajectory code. This code is capable of solving Poisson equation in 3-D for any arbitrary electrode shapes. The resulting beam shape, spread in velocities and the effect of various cathode geometries on the sheet electron beam will be discussed.
This work is supported by AFSOR.

- ☐ Prefer Poster Session
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(specify order)

- ☐ Special Facilities Requested
(e.g., movie projector)

- ☐ Other Special Requests

Submitted by:

Altan Ferendeci

(Signature of APS Member)

(Same Name Typewritten)

(Address)

This form, or a reasonable facsimile, plus TWO XEROX COPIES, must be received by NO LATER THAN NOON, Monday, July 10, 1989 at the following address:

Saralyn Stewart
Institute for Fusion Studies
RLM 11.234
The University of Texas at Austin
Austin, TX 78712
Telephone: (512) 471-4378

Abstract submitted for the Thirty-first Annual Meeting
Division of Plasma Physics
November 13-17 1989

Category Number and Subject _____

☒ Theory ☐ Experiment

3-D Numerical Solution of Particle Trajectories with Arbitrary Electrode Geometries. ALTAN M. FERENDECI, University of Cincinnati.-- A numerical 3-D Fortran code is developed to simulate electron trajectories in electron guns. Once necessary cathode and accelerator electrode shapes are found by using various synthesis techniques, these are used in a bit-map program to calculate fractional distances from regular mesh points to the boundary electrodes. Once the 3-D Laplace equation is solved, the relativistic electron trajectories subjected to the electric fields derived from the potentials and the externally inputted magnetic fields are found. The resulting charge is then used to solve the Poisson equation. The process is repeated until convergence of trajectories occur. The number of 3-D mesh points are limited by the memory space of the computer. The bit-map program run using a MacII requires close interaction between the user and the computer. The program also calculates and plots the spread in velocities at the entrance to the interaction region. The code can be applied to other particle trajectory calculations. This work is supported by AFSOR.

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Submitted by:

Altan M. Ferendeci

(Signature of APS Member)

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The University of Texas at Austin
Austin, TX 78712
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NUMERICAL SOLUTION OF LAPLACE EQUATION IN THREE DIMENSIONS FOR A PIERCE TYPE ELECTRON GUN

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(Received for publication 27 June 1989)

Abstract—In this paper, as part of a three-dimensional ribbon beam gun design, Laplace equation is solved in 3D for the case of boundaries which can be represented as simple surfaces which are subjected to specified voltages. Curve fitting by the least squares method is used to characterize the accelerator electrode shape as an extension of the 1D laminar electron flow in a Pierce type electron gun. The method involves solution of governing equations by an iterative finite difference technique. Modification of the standard Leibmann procedure is used to greatly increase the rapidity of convergence. The technique of handling irregular boundary points is considered in detail.

INTRODUCTION

Analytical solution of the Poisson equation, possible under very restricted simple cases, generally involves applying the necessary boundary conditions as well as specifying the electrical charge through proper flow equations. Two-dimensional solutions can be obtained provided there are simple metallic boundaries and uniform charge. As the geometry becomes complicated, complexity of the analytical solutions increases in proportion and sometimes even becomes impossible. On the other hand, electrical characteristics of many electron devices are based on solution of the Poisson equation. One of the major difficulties associated with that task is the specification of boundary conditions appropriate to the problem. Only for simple geometries, Neumann boundary conditions can be relatively easily incorporated into the numerical solution of the problem.

Numerical techniques may be used to overcome some of the difficulties associated with the analytical methods. A finite difference or finite element technique may be used in the numerical solution of the problem. In either case, proper boundary identification is required, and the relative distances from the boundary points to the regular mesh points or the vertices of the triangles must be known before the main program can be carried out.

In designing an electron gun, it is necessary to determine the proper electrode shapes so that the resulting electron beam will have such desired characteristics as laminar flow or space charge limited operation. Elegant synthesis techniques have been developed to find the required electrode shapes possessing cylindrical symmetry [1]. With these techniques, Magnetron Injection Guns (MIG) have been successfully designed and tested [2]. Before the gun is actually built, it is simulated using a numerical code to verify the validity of the desired gun parameters [3].

Rotating electron ribbon beams are needed for realization of an axially grooved rectangular gyrotron operating at the higher harmonics of the cyclotron frequency [4] and the quasi-optical gyrotron [5]. These beams require three-dimensional solution of the Poisson (or Laplace) equation for the simulation of electron trajectories.

The model of the electron gun used in this study is the 3-D adaptation of the modified Pierce electron gun (MPEG), which contains four essential components as shown in Fig. 1 [6]. Electrode

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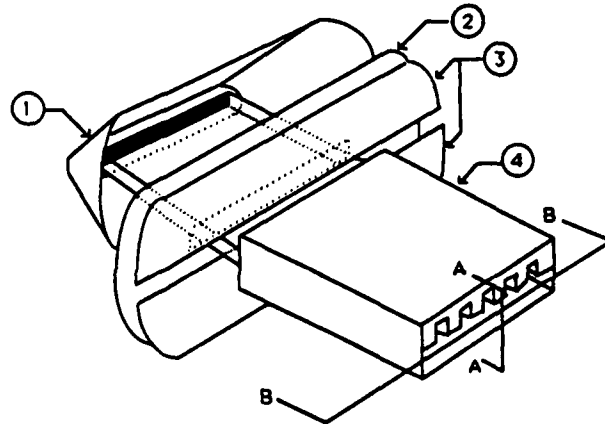


Fig. 1. General schematic of a 3D Pierce gun. (1) Focusing electrode, (2) accelerating electrode, (3) kicker electrodes and (4) interaction region.

No. 1 consists of a flat (or curved) cathode having a height of $2h_0$ and a width of $2w_0$. The surrounding focus electrodes are made up of two inclined planes and two half cones, all at the same zero potential. The planes and the half cones make an angle α with the z axis. This angle is considered a variable in the design of the gun. Electrode No. 2 consists of a metallic accelerating plate that has a special curvature to maintain laminar electron flow of the electrons. It also contains an opening (an aperture) allowing the electrons to flow through to the interaction region.

The No. 3 electrodes, called "kicker electrodes," consist of two straight plates and are separated by a distance of $2h_0$ to form a window at the center. Each of these electrodes is subjected to a different potential which produces an electric field giving an initial kick to the electrons in the transverse plane so that they can initiate rotational motion. If there is an increasing magnetic field beyond the kicker electrodes, the perpendicular energy of the electrons increases and their orbits get smaller, finally reaching their intended orbits before entering the interaction region. In the problem considered here, the interaction region is replaced by the flat electrode No. 4, which is used as a collector for the electrons. Figure 2 shows the projections of the pertinent distances and parameters of the gun. The outer edges of electrodes Nos 2, 3 and 4 have the same circular shape as of the focus electrodes and are independent of the z coordinate. Thus the Neumann boundaries become a two-dimensional problem for the given geometry.

ACCELERATOR ELECTRODES IN RECTANGULAR COORDINATES

A finite difference technique is used in the simulation of the electron gun. In solving Poisson equation in 3D, boundary identification is a priority. By using a synthesis technique, appropriate electrode shapes can be found so that the resulting structure will satisfy the requirements for the proper operation of the device. In general, because of the occurrence of irregular boundaries, one must also determine the distances between these boundary points and the regular mesh points. To accomplish this, it is necessary to establish the precise shapes and the location of metallic boundaries where potentials are applied. This can be done relatively easily, provided the surfaces are simple and they can be represented by analytical equations.

The electrode shapes for the focusing and the accelerating electrodes are found from the extension of the analysis based on the one-dimensional Pierce gun [7]. In this model, two infinitely long parallel plates are separated by a distance " d ." Electrons are emitted from one of the electrodes and move toward the second electrode under the application of an accelerating potential V_0 . When the resulting equations of motion of the electrons are solved with the help of the Maxwell and energy conservation equations, the potential, charge and velocity variation as a function of the distance x is found. In particular, the potential varies as $F(x) = Ax^{4/3}$. The problem can then be reduced to a finite geometry if a set of electrode shapes is found so that the potential variation at the boundary of the finite beam is the same as the potential variation of the infinitely large beam.

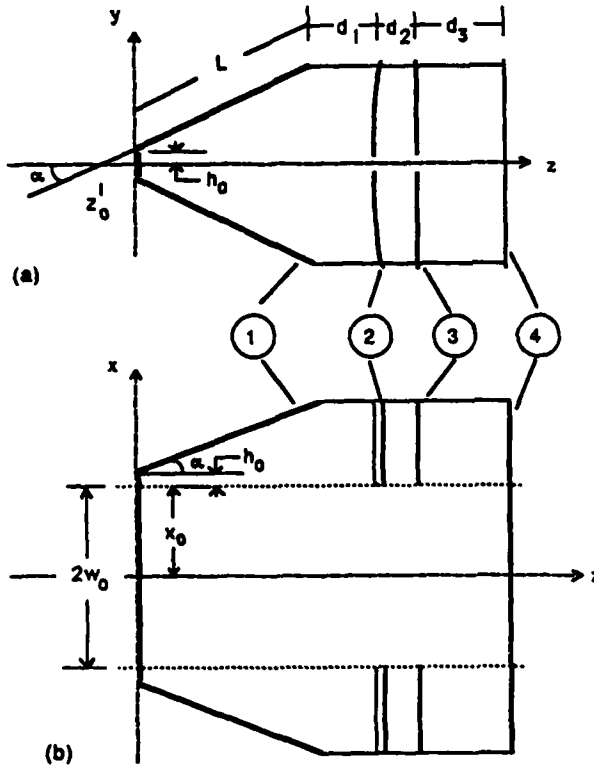


Fig. 2. Cross sectional diagrams and the corresponding parameters used in the program. Cross sections: (a) AA, and (b) BB. Interaction region (No. 4 in Fig. 1) is replaced by a flat plate.

This synthesis procedure is carried out by means of conformal transformation

$$W = AZ^{4/3} \quad (1)$$

where $W = u + jv$ and $Z = x + jy$ are complex numbers

$$W = (u + jv) = (x + jy)^{4/3} = r^{4/3} e^{j4\theta/3}. \quad (2)$$

It is clear that at $y = 0$, the real part of W is

$$u = x^{4/3} \quad (3)$$

and is the same as the required potential. Furthermore, we see that $\partial w / \partial z$ exists and is unique except at $z = 0$.

Thus u is a suitable potential function for this problem. The real part of equation (2) gives the potential distribution. Equipotentials are found by setting u equal to a constant V_0 where V_0 is an arbitrary potential assigned to an equipotential surface. Consequently equation (2) gives

$$V_0 = r^{4/3} \cos\left(\frac{4\theta}{3}\right). \quad (4)$$

Setting $V_0 = 0$, the angle θ is found to be 67° . This is the angle α that the two flat plates attached to the cathode make with the horizontal axis as shown in Fig. 2.

To find the shape of the accelerating electrode, equation (4) is solved for r

$$r = \frac{V_0^{3/4}}{\cos^{3/4}\left(\frac{4\theta}{3}\right)}. \quad (5)$$

For various values of θ , corresponding values of r can be found.

In finding the equation of the electrodes in an analytical form $y = f(x)$, equation (5) is too complex to transform it into rectangular coordinates. To surmount this problem, curve fitting by the method of least squares is adapted. Equation (5) is solved for x and y in terms of the angle θ , and for different values of angle θ , corresponding x and y are found. The resulting points are represented by a function of the form

$$y = G(x) = a_1(x - x_{00})^{a_2} \quad (6)$$

where x_{00} , a_1 and a_2 are real numbers, and x_{00} can be found by setting the angle θ equal to zero in equation (5).

Using the least squares method, the spatial dependence of the accelerator electrodes can be written in the following form

$$G(x) = 8.813(x - 9.457)^{0.571}. \quad (7)$$

To find the equation for the two conical plates, it will be assumed that they have the same inclination angle α as the two flat focusing plates above and below the cathode. The equation for a general cone with the origin shifted to y_0 and $-z_0$ can be written as

$$\frac{x^2}{a_0^2} + \frac{(y - y_0)^2}{b_0^2} + \frac{(z + z_0)^2}{c_0^2} = 0 \quad (8)$$

where a_0 , b_0 and c_0 are constants. For a cone of semicircular shape (Fig. 2)

$$z_0 = \frac{x_0}{\tan \alpha}, \quad (a)$$

$$c_0 = L \cos \alpha + z_0, \quad (b)$$

and

$$R_1 = a_0 = b_0 = c_0 \tan \alpha = (L \cos \alpha + z_0) \tan \alpha. \quad (c) \quad (9)$$

It should be emphasized that for the present analysis, the semicircular edges of the accelerator and the kicker electrodes have the same circular arc as the two ends of the focusing cone electrodes. Note that in the program, provisions are made so that, if necessary, all the electrode parameters, including the angle α , can be changed.

FINITE DIFFERENCE EQUATIONS IN 3D FOR DIFFERENT MESH SIZES

The finite difference method is sufficiently well known in two dimensions [8], and little is necessary to extend it to three dimensions. There are two main categories of boundaries to be considered, namely regular and irregular.

Regular boundary conditions

The potential at a point in space can be found by the potentials surrounding that point. In three dimensions and with different mesh sizes of h_x , h_y and h_z [Fig. 3(a)], the central potential, including the charge density ρ , can be written as

$$\begin{aligned} \Phi(x, y, z) = \frac{1}{2[h_x^2 h_y^2 + h_x^2 h_z^2 + h_y^2 h_z^2]} \{ & h_y^2 h_z^2 [\Phi(x + h_x, y, z) + \Phi(x - h_x, y, z)] \\ & + h_x^2 h_z^2 [\Phi(x, y + h_y, z) + \Phi(x, y - h_y, z)] \\ & + h_x^2 h_y^2 [\Phi(x, y, z + h_z) + \Phi(x, y, z - h_z)] \frac{\rho}{\epsilon_0} h_x^2 h_y^2 h_z^2 \}. \end{aligned} \quad (10)$$

This is called the 7-point finite-difference equation in 3D for regular boundary. For the special case when $h_x = h_y = h_z = h$ and $\rho = 0$, it reduces to the following simple, well known form

$$\begin{aligned} \Phi(x, y, z) = \frac{1}{6} [& \Phi(x + h, y, z) + \Phi(x - h, y, z) + \Phi(x, y + h, z) + \Phi(x, y - h, z) \\ & + \Phi(x, y, z + h) + \Phi(x, y, z - h)]. \end{aligned} \quad (11)$$

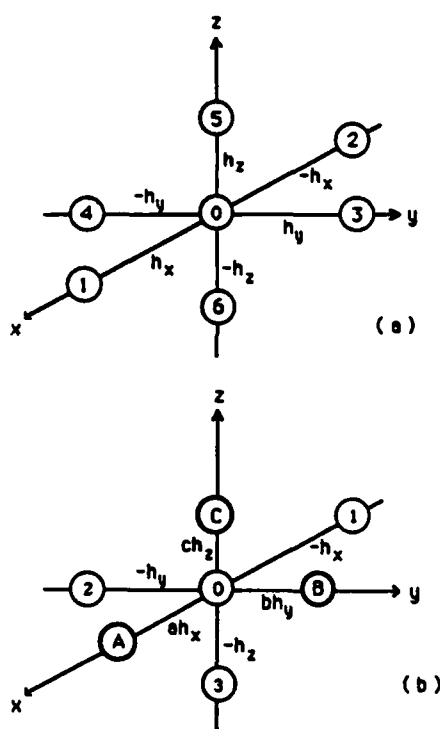


Fig. 3. (a) Regular boundary and (b) irregular boundary points used to calculate the potentials. A, B and/or C may lie on a metallic boundary. a , b , and c are, respectively, the fractional distances from the regular mesh point at O to the boundary points A, B and C.

Irregular boundary conditions

Charge is usually present at the central regions of the gun where regular boundary points apply. Near the metallic boundaries, ρ can be set to zero without changing the character of the problem. Given boundary conditions of the Dirichlet type, the potential is known along the curved boundary on which points A, B and C lie. For simplicity, the subscript notation will be temporarily omitted, and the relevant interior grid points will be labeled as 0, 1, 2 and 3 as indicated in Fig. 3(b). Using partial expansion and combining the terms, the potential at the central point can be written as

$$\Phi(x, y, z) = \frac{abc}{abh_x^2h_z^2 + ach_x^2h_z^2 + bch_y^2h_z^2} \left[\frac{h_y^2h_z^2}{a+1} \phi_1 + \frac{h_x^2h_z^2}{b+1} \phi_2 + \frac{h_x^2h_y^2}{c+1} \phi_3 \right. \\ \left. + \frac{h_y^2h_z^2}{a(a+1)} \phi_A + \frac{h_x^2h_z^2}{b(b+1)} \phi_B + \frac{h_x^2h_y^2}{c(c+1)} \phi_C \right]. \quad (12)$$

Here a , b and c are the fractional numbers in the x , y and z directions, respectively, and need to be evaluated for each irregular point.

Neumann boundary conditions

Neumann boundary conditions occur in the case of gaps between electrodes. Fortunately for the problem considered here, the distances between the edges of the different electrodes fall on simple planes. At the top and bottom they are flat planes, and at the edges they are circular, but in both cases their boundary lines are equidistant from each other. For the straight portions of the space between the edges of the plates, Neumann boundary conditions, i.e. the zero normal derivative of the potential, can easily be taken care of in the program. This is done by equating the potential at the boundary to the neighboring interior point. Both of these points lie in the direction of the normal to the boundary surface.

An interesting difficulty is encountered when mesh points lie close to the curved boundaries as shown in Fig. 4(a). For example, in considering point A, the finite difference expression includes

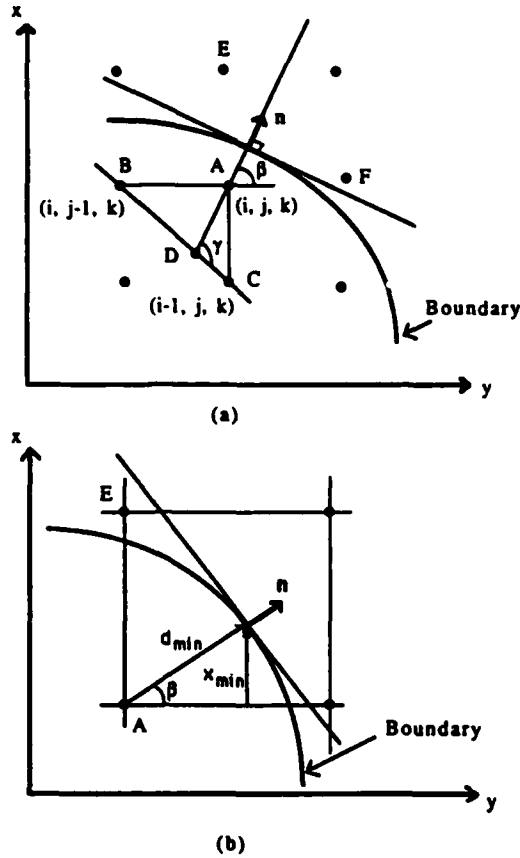


Fig. 4. Neumann boundary over an open ended curve. (a) Potential at D is extrapolated from potentials at B and C. (b) Normal n is located by finding the minimum distance from the mesh point A to the boundary curve.

points E and F. Now, however, no such boundary values are available. Fortunately, because of the given geometry of the problem, the Neumann boundaries can still be treated as a two-dimensional case. One way of doing this has been presented by Fox [9]. Irregular points can be handled by deriving special triangular finite difference equations. Using linear interpolation and introducing point D as in Fig. 4(a), the potential at D can be written as

$$\phi_D = \phi_B + (\phi_C - \phi_B) \frac{BD}{BC} \quad (13)$$

or

$$\phi_D = \frac{DC}{BC} \phi_B + \frac{BD}{BC} \phi_C. \quad (14)$$

Also, to the same approximation

$$\left(\frac{\partial \phi}{\partial n} \right)_A = \frac{\phi_A - \phi_D}{AD}. \quad (15)$$

The Neumann boundaries represent gaps between surfaces that must be chosen so that the normal component of the field is zero. Hence

$$\phi_A = \phi_D. \quad (16)$$

Substituting equation (16) into (14) results in

$$\phi(i, j, k) = \frac{DC}{\sqrt{2}} \phi(i, j-1, k) + \frac{BD}{\sqrt{2}} \phi(i-1, j, k). \quad (17)$$

Now the problem is to eliminate the parameters DC and BD . Using the law of sines and referring to Fig. 4(b), one can write

$$\frac{BD}{\sin \beta} = \frac{BA}{\sin \gamma} = \frac{1}{\sin\left(\pi - \frac{\pi}{4} - \beta\right)} = \frac{\sqrt{2}}{\sin \beta + \cos \beta}$$

where β is the angle that the normal vector \mathbf{n} makes with the y axis. BA and BC are set to one. After some mathematical manipulations, one obtains

$$\Phi(i, j, k) = \frac{1}{1 + \tan \beta} \Phi(i, j - 1, k) + \frac{\tan \beta}{1 + \tan \beta} \Phi(i - 1, j, k), \quad (18)$$

which represents a triangular finite difference equation for the general 2D Neumann problem for equal mesh points.

In this case, $\tan \beta$ can be calculated [see Fig. 4(b)] from

$$\tan \beta = \frac{x_{\min}}{\sqrt{d_{\min}^2 - x_{\min}^2}} \quad (19)$$

where d_{\min} is the shortest distance from the mesh point A to the boundary and x_{\min} is the minimum distance from the x -location of point A to the intersection of normal with the tangent on the boundary.

NUMERICAL SOLUTION AND RESULTS

The geometry of the problem is a set of curved planes which intersect with the x , y and z axes and produce fractions of the grid points a , b and c . Each fraction can be less than or equal to one unit. Before solving the equations, these fractional distances are calculated for all the mesh points. Only those that lie close to the boundary points will have fractional values. The results are stored in a file and called back when needed in the main program.

The general approach is then to perform iterations over all grid points, starting from the nearest nonzero potentials and moving toward the interior points using the appropriate finite-difference approximation and successive over relaxation (SOR) method. The main objective of the relaxation method is to relax the largest residual as close to zero as possible by changing the corresponding nodal potentials by proper amounts. For a 3D system, it is necessary to change the nodal potentials by one-sixth of the change effected in the residual [10].

To make the most effective use of SOR, it is important to use the optimum value of ω where ω is a convergence factor determining the degree of control over the rate of convergence. The value of ω is dependent in a complex manner upon the boundary shape, boundary conditions and number of nodes, so that its theoretical evaluation [11] is extremely difficult and indeed is possible only for a few simple boundary shapes [12-14]. Since the boundaries in the present problem are also extremely complicated, it is not theoretically possible to evaluate the optimum value of the convergence factor.

In the present program, optimum convergence factor ω in the range 1-2 was tried [15]. As ω is varied, beginning with 1.0, each time the program is run, the number of iterations is recorded to reach a specified minimum residual. In case the results converge too slowly, the calculation is stopped, ω is increased by a fraction and calculation is restarted. The best choice is the value of ω that leads to a minimum number of iterations. In the present program, $\omega = 1.75$ reduced the number of iterations from 332 to 57 for a residue of 0.001.

Throughout this study, potential distribution $\Phi(x, y, z)$ in the interior regions of a 3D EG has been simulated by a computer. The parameters used in the sample calculations are given in Table 1. Equipotential lines have been plotted on a Tektronik 4110 plotting terminal. Some of these equipotential lines are shown in Figs 5-7 at different planes. The validity of the resulting solutions is tested by the numerical print-out of the potential values and by visual inspection of the plots. In the present electron gun design, all the dimensions of the gun, as well as the distances, including the inclination angle α , are considered variables. By varying especially the angle α around design

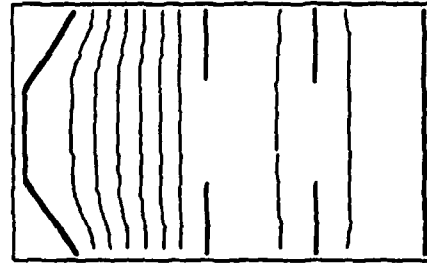
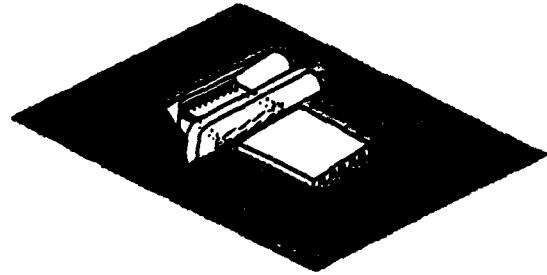
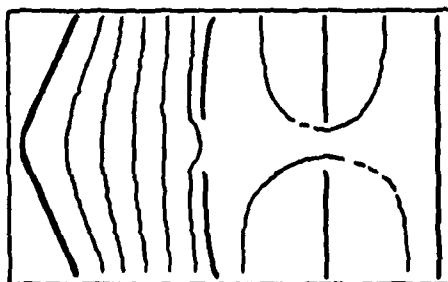
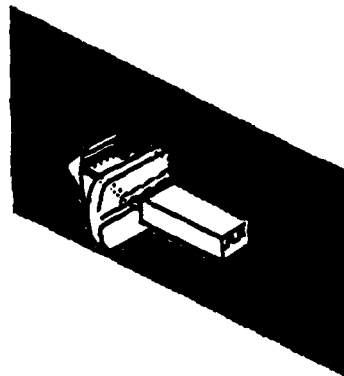
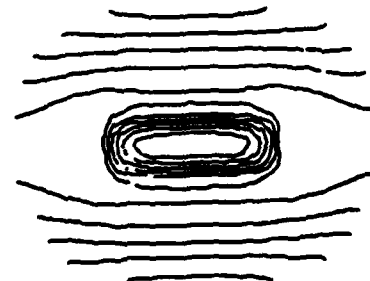
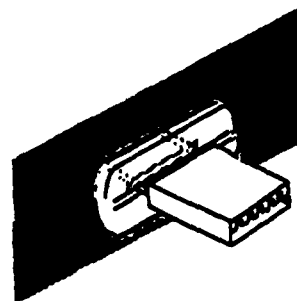
Fig. 5. Equipotential lines at $y = 0$ in the (x, z) plane.

Table I	
Parameter	Value
α	67°
$2h_0$	5.0 mm
$2w_0$	45.0 mm
d_1	10.7 mm
d_2	7.0 mm
d_3	20.0 mm
V_1	0.0 V
V_2	20.0 kV
V_{Upper}	15.0 kV
V_{Lower}	25.0 kV
V_4	20.0 kV

angle of 67° , one can automatically change other parameters of the focus and the accelerating electrodes to allow for fine-tuning of the fields and eventually, well focused electron ribbon beams.

CONCLUSION

In this paper, the problem of non-symmetry was handled by solving Laplace equation in 3D subject to specified boundary values of voltages on electrodes and normal fields on open curves. Curve fitting by the least squares method has been used to determine the precise accelerator electrode

Fig. 6. Equipotential lines at $x = 0$ in the (y, z) plane.Fig. 7. Equipotential lines at (x, y) plane close to the left of the accelerating electrode. The fringing in the middle is due to the window on the electrode.

shapes needed to produce a laminar electron flow in the gun. A program has been developed which is coded in FORTRAN 77 to run in a VAX computer, and successful results have been obtained.

The program can be extended without major modification to solve Poisson equation in 3D. Since the charge is confined to the interior regions of the gun, inclusion of charge will affect only equation (10). If the equations of motion and the space charge forces between electrons are included as well, complete gun design can be accomplished. For a given geometry, calculation of the electron trajectories will provide new fields to be substituted back into the trajectory equations. After a few iterations, the complete set of electron trajectories can be determined. The plot of these trajectories will indicate the nature of the electron flow, their distribution and the spread in their velocities.

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